Geometry
Big Ideas Chapter 6 Study Guide -Relationships Within Triangles

Definitions

Equidistant: describes two points in which each are the same distance from a third point. e.g. On a segment, the midpoint is equidistant from the endpoints.

Concurrent: three or more lines, rays, or segments intersect in the same point.

Circumcenter: the concurrent point of the perpendicular bisectors of a triangle.

**Incenter**: the concurrent point of the angle bisectors of a triangle.

Median: the segment that connects a triangle's vertex to the midpoint of the opposite side.

Centroid: the concurrent point of the medians of a triangle.

Altitude: the segment drawn from a vertex of a triangle to the line containing the opposite side and forming a perpendicular.

Orthocenter: the current point of the altitudes of a triangle.

Midsegment: a segment that connects the midpoints of two sides of the triangle.
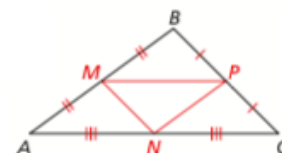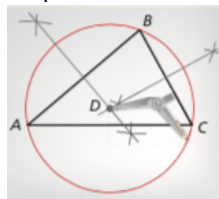
Theorems and Postulates

- **Perpendicular Bisector Theorem (and Converse)**: In a plane, a point lies on the perpendicular bisector of a segment if and only if it is equidistant from the endpoints of the segment.
- **Angle Bisector Theorem (and Converse)**: A point lies on the bisector of an angle if and only if it is equidistant from the two sides of the angle.
- **Circumcenter Theorem**: The circumcenter of a triangle is equidistant from the vertices of the triangle.
- **Incenter Theorem**: The incenter of a triangle is equidistant from the sides of the triangle.
- **Centroid Theorem**: The centroid of a triangle divides the medians into a one-third, two-third distance ratio.
- **Triangle Midsegment Theorem**: The segment connecting the midpoints of two sides of B a triangle is parallel to the third side and is half as long as that side.
- **Triangle Longer Side Theorem**: If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.
- **Triangle Larger Angle Theorem**: If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.
- **Triangle Inequality Theorem**: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- **Hinge Theorem**: If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.
- **Converse of the Hinge Theorem**: If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

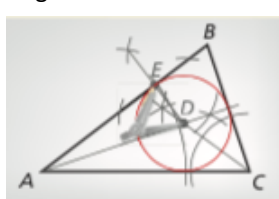## How to Write an Indirect Proof (Proof by Contradiction)

1. Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true.
2. Reason logically until you reach a contradiction.
3. Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.
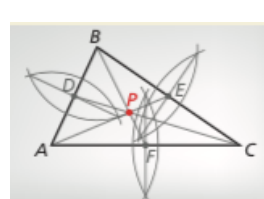
Constructions

Circumcenter (P. 312)	Incenter (P. 314)	Centroid (P. 320)
Circumscribing a circle	Inscribing a circle	Center of Mass
Perpendicular bisectors	Angle bisectors	Medians

1) Consider the points P(2, 0), A(-4, 2), B(0, -6), and C(6, -3). Show that P is on the bisector of $\angle ABC$.

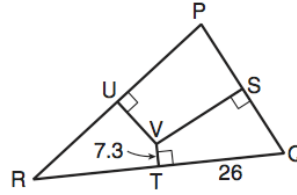2) $\overline{VS}$, $\overline{VT}$, and $\overline{VU}$ are perpendicular bisectors of the sides of $\triangle PQR$. Find the circumference of the circle that can be circumscribed about this triangle.

3) Find the orthocenter of the triangle with the vertices X(-5, 4), Y(2, -3), Z(1, 4).

4) $GB = 12\frac{2}{3}$ and $CD = 10$. Find each length.

FG = _____
BF = _____
GD = _____
CG = _____

5) The midpoints of the sides of a triangle are A(-6, 3), B(2, 1), and C(0, -3). Find the coordinates of the vertices of the triangle.

6) The diagram shows horseback riding trails. Point B is the halfway point along path $\overline{AC}$. Point D is the halfway point along path $\overline{CE}$. If riders travel from A to B to D to C, and then back to A, how far do they travel?.
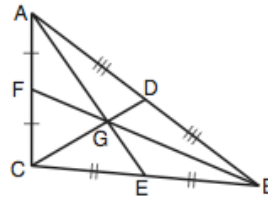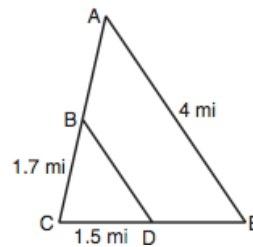

7) Renaldo plans to take a plane from Atlanta to London (4281 miles). On the return, he will fly back from London to New York City (3470 miles) to visit his aunt. Then Renaldo heads back to Atlanta. Atlanta, New York City, and London do not lie on the same line. Find the range of the total distance Renaldo could travel on his trip.

8) In $\triangle DEF$, $m\angle D = (x + 25)°$, $m\angle E = (2x - 4)°$, and $m\angle F = 63°$. List the side lengths and angle measures of the triangle in order from least to greatest.

9) Write an indirect proof that the angle measures of a triangle cannot add to more than 180°. Use the diagram, the Exterior Angle Theorem, and the Linear Pair Theorem.